

Linear interpolation example

Today's date is December 5, 2005. A bank needs to determine a USD Libor rate with a maturity of January 19, 2006, which is approximately 1½ months from today. Rate source is BBA Libor. There is no current Libor quote available for the required maturity, however, so it is necessary to estimate the unknown rate by means of linear interpolation.

Solution. Let R_n denote the unknown Libor rate, with maturity n . The closest *designated maturities* available are one month (R_1) and two months (R_2), which bracket the unknown rate from above and below. Linear interpolation, which calculates the unknown rate as if it lies on a straight line between the two rates, proceeds as follows.

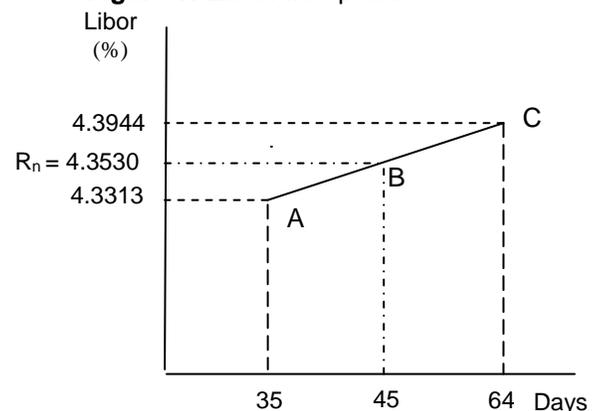
Today's date:	December 5, 2005
Maturity date of unknown rate:	January 19, 2006
Days to maturity of unknown rate (t_n):	45
Today's 1-month BBA Libor (R_1):	4.3313%
Maturity date of R_1 :	January 9, 2006 ¹
Days to maturity of R_1 (t_1):	35
Today's 2-month Libor (R_2):	4.3944%
Maturity date of R_2 :	February 7, 2006
Days to maturity of R_2 (t_2):	64

Figure 1 shows the relationship between the two rates and days to maturity. Linear interpolation assumes that the unknown rate (R_n) lies on the line (AC) between the two known rates. Because AC is linear, that is, a straight line, the slope of the line (AB) connecting R_1 and R_n is the same as the slope of line AC. Using the "rise over run" formula for the slope of the line, we solve for R_n as follows:

$$\begin{aligned} R_n &= R_1 + \frac{R_2 - R_1}{t_2 - t_1} \times (t_n - t_1) \\ &= 4.3313\% + \frac{4.3944\% - 4.3313\%}{64 - 35} \times (45 - 35) \\ &= 4.3313\% + 0.00218\% \times (10) = 4.3530\% \end{aligned}$$

The interpolated rate is 4.3530%, which lies between the two known rates.

Figure 1: Linear interpolation



¹ The reset date for BBA USD Libor rates is normally two days after the fixing date, so the maturity date for 1M Libor fixed on December 5 would normally be January 7. January 7, 2006, however, falls on a Saturday. According to the Modified Following business day convention, the maturity date moves forward to the next "good" business day so long as it is in the same month.

General formula. The following is a derivation of the general version of the above equation. Days to maturity corresponding to a designated maturity should be adjusted as discussed in the following section. Given two known rates and their maturities:

- R_1 = Known rate with shorter designated maturity
- t_1 = Days to maturity of R_1
- R_2 = Known rate with longer maturity
- t_2 = Days to maturity of R_2
- R_n = Unknown rate between R_1 and R_2
- t_n = Days to maturity of R_n

find unknown rate R_n with known maturity t_n ($t_1 < t_n < t_2$) based on the assumption that the slope of line AB in Figure 2 is equal to the slope of line AC as follows.

$$\begin{aligned} \frac{R_n - R_1}{t_n - t_1} &= \frac{R_2 - R_1}{t_2 - t_1} \\ \Rightarrow R_n - R_1 &= \frac{R_2 - R_1}{t_2 - t_1} \times (t_n - t_1) \\ \Rightarrow R_n &= R_1 + \frac{R_2 - R_1}{t_2 - t_1} \times (t_n - t_1) \end{aligned} \quad (1)$$

which is a general formula of linear interpolation.

Another commonly used—and mathematically equivalent—version of the linear interpolation formula is the following:

$$R_n = \frac{[R_1 \times (t_2 - t_n)] + [R_2 \times (t_n - t_1)]}{(t_2 - t_1)} \quad (2)$$

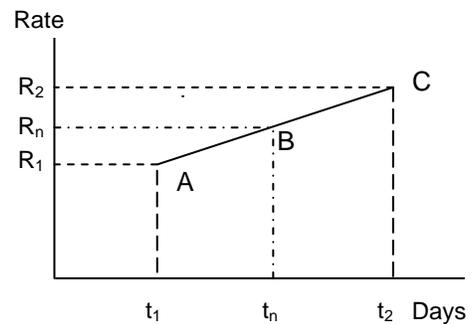
In the preceding example, the solution would work as follows:

$$R_n = \frac{[4.3313\% \times (64 - 45)] + [4.3944\% \times (45 - 35)]}{(64 - 35)} = 4.3530\%$$

which is the same result as that obtained using the formula (1).

Determining days corresponding to designated maturity. The linear interpolation formula requires that maturity be expressed in days, yet designated maturities are often expressed in months.

Figure 2 Linear interpolation (general)



Libor and Euribor rates are an example. For example, the USD Libor rate fixed on October 21, 2009, will become effective (i.e., reset) on October 23 and will mature on Monday, November 23. Because Monday is a “good” Business Day in both New York and London, no further adjustment is required. But the USD Libor rate fixed on October 20, 2009, will reset on October 22 but will mature not on November 22, which is Sunday, but on November 23, the next good Business Day. It is the maturity dates of the Libor rates that should be used to determine the number of days corresponding to designated maturities.

Linear interpolation is generally used to determine payments for a particular Calculation Period. Because the Calculation Period is the interval between two Period End Dates, days to maturity for rates used in interpolation should be adjusted using the same Business Days and Business Day Convention as those used for Period End Dates. That means the following:

- If the confirmation for a transaction specifies Business Days or Business Day Convention or both, use the adjustment specified in the confirmation (unless Period End Dates are unadjusted; see below).
- If the confirmation does not specify Business Days, follow the guidance in Sections 1.4 through 1.6 of the 2006 ISDA Definitions. So for USD Libor, for example, one would use New York and London Business Days (Section 1.6(c)).
- If the confirmation does not specify Business Day Convention, use Modified Following (Section 4.10).
- If the confirmation specifies No Adjustment (Period End Dates are unadjusted), use the number of days corresponding to the unadjusted dates.

For more detail on the above guidance, see the Best Practice Statement at <http://www.isda.org/marketpractice/pdf/Linear-Interpolation-BP.pdf>